Collaborative strategies to teach set theory using digital systems

J. Santander Castillo, Y. Ramírez Chocolatl, V. M. Castañeda Téllez, M. Ramírez Jaime

***Resumen*. Se diseñó un nuevo método para enseñar teoría de conjuntos para estudiantes de ingeniería y estudiantes de ciencias de la computación o ingeniería computacional, está basado en aprendizaje colaborativo donde los estudiantes trabajan en equipo y cada uno de los integrantes tiene asignado un rol diferente. Cada equipo se divide en tres estudiantes clasificados como el "verificador", el "modelador" y el "constructor".**

Como primer paso, el método permite construir una equivalencia entre los conceptos básicos de la teoría de conjuntos como unión, intersección y complemento con los conceptos básicos de lógica matemática como disyunción, conjunción y negación. Posteriormente se plantea un problema de teoría de conjuntos, se inicia el modelado en lógica digital (realizado por todos los "verificadores") y se diseña la tabla de verdad con todos los resultados verdaderos y falsos posibles (diseñada por todos los "modeladores"). Finalmente se construye el diagrama de compuertas ("por los constructores") y el circuito correspondiente en un protoboard (juntando los resultados obtenidos por el "verificador", "modelador" y "constructor" de un equipo) donde los estudiantes pueden modelar el problema propuesto de teoría de conjuntos usando lógica digital.

El método propuesto fue aplicado para enseñar teoría de conjuntos a estudiantes de ingeniería en sistemas y biotecnología logrando un mejor aprendizaje de esta área de las matemáticas para los estudiantes.

***Palabras Clave*. *Teoría de conjuntos, lógica digital, circuitos TTL, aprendizaje colaborativo*.**

***Abstract*. We design a new method for teach set theory for engineer students and computer science engineer, this method is based in collaborative education where the student make a team and each member have a different role. Each team is divided in three students called the "checker", the "modeller" and the "builder"**

The first step is to build equivalence between the basic concepts of the set theory like union, intersection and complement with the digital logic basic concepts like disjunction, conjunction and negation. Later, the students analyze a set theory problem making the digital logic equivalent(made for all "checkers"); the "modellers" design the truth table with all the possibilities, in this table they have the combination of true or false values. Finally, the "builders" build the equivalent circuit using logic TTL gates and on the breadboard make the result using all information obtained by a team (checker, modeller and builder); the team can model problems of the set theory using digital logic. This method was used to teach set theory to computer system engineering and biotechnology engineering students, they have a better knowledge of this area of the mathematics.

***Keywords. Set theory, digital logic, TTL circuits, collaborative education*.**

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I. INTRODUCTION

Teaching model at the Polytechnic Universities in Mexico are the model teaching competencies, this model allow to students teaching skills for to work into their future jobs, this model allow to students play the role of their future. The teacher often includes knowledge, skills and attitudes to students. The tools and techniques deployed are regular meetings to review of competences, peer review, video analysis, reports, portfolio, written narratives and others [1-4].

The collaborative education is not only a technique for the classroom, is a personal live philosophy. In all meetings is necessary that each person collaborate and contribute as a member of the community. For that the members of the community have individual's

abilities inside of this group. In a collaborative group exist a shared authority and the members of this community accept their responsibilities and the actions of each member and take decisions like a group [2-10].

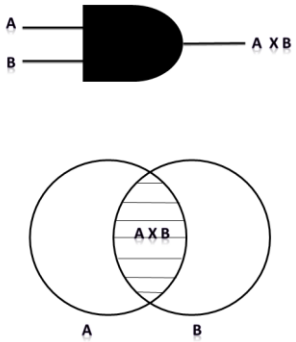
The successful of the collaborative model is the cooperation between individual people in contrast with the competences education; here individuals are better than others in their same group. The students in the collaborative education learn to live like a social community and each individual act like a family where each one has a role in his home. The students learn a new form of live and a new form to live with other people.

The formal collaborative education need stimulate attitude and values, develop a critical thinking and of course the ability of play different roles. For that, the students require learn to listen to the other members of their community and know their points of view. The teacher requires instruct to his students for analyze the new knowledge and distribute a role for each student during its learning [1].

In the engineering education for many time was chalk and talk education [11, 12], the engineer has the necessity of make a team work, require skills and practice. Other option was a problem based education or project based learning [2, 13-15].

On another hand, set theory and all the relationship with the mathematics is difficult for the most of the students, they cannot be the close relation between mathematics and engineering.

In conclusion, the engineer needs to work in form collaborative taking a role inside his group. For that we describe a new method for to learn set theory using the previous knowledge of circuit theory that is most recognized for the engineer.



Truth Table AND		
A	B	A x B
0	0	0
0	1	0
1	0	0
1	1	1

Figure 1.- AND gate and his equivalent in set theory, the conjunction with intersection**.**

II. METHODOLOGY

The students of engineering learn set theory after of the circuit theory, so they have knowledge for make a little circuits with logic gates and TTL circuit like AND, OR and NOT gates.

Our model begins with a correspondence essential between set theory and logic. The first correspondence is analyzed of the definition of intersection [16]. This say:

$$A \cap B = \{x \in A \wedge x \in B\} \quad (1)$$

Where x is an element that belongs to set A and belongs to set B

So, in the definition is implicit that the intersection involves the preposition "and".

The figure 1 shows this correspondence and is explained in this form, we have two sets denominated A and B, if exist a relation named "intersection" between this sets then we have one element that belong to both sets, this belong to set A and to set B, this occurs only if is true that exist this element in two sets. This analysis in logic mathematic means that I can change INTERSECTION for AND gate and put an equivalent truth table. So, only if exist the element in the two sets (and exist conjunction between them) then the truth table has a "true" value. Elsewhere, if there is not a relation between the sets then the truth values are "false", a false value in the table means that the element does not exist in the set. In logic circuit the AND gate is "true" when both values are true, in other cases are false.

The second correspondence is about of union definition [16]. This say:

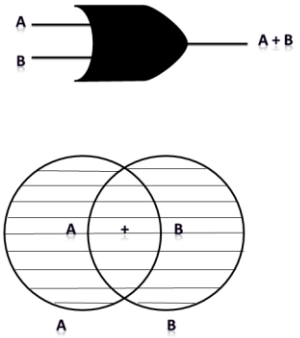
$$A \cup B = \{x \in A \vee x \in B\} \quad (2)$$

Where x is an element that belongs to set A or belongs to set B.

So, in the union definition is implicit the preposition "or".

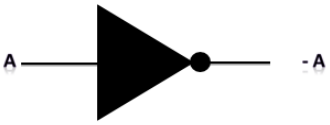
The figure 2 shows that we have two sets denominated A and B, if exist a relation named "union" between this sets then is true that we have one element that belong or A or B or both sets.

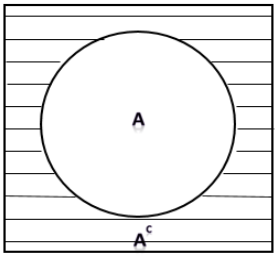
This analysis in logic mathematic means that we can change UNION for OR gate and put a truth table equivalent. So, if exist one element that belong to some set, in the table we have a true value (and exist the disjunction between them) then the truth table have a "true" value if exist the element in some sets, this only will be "false" if the sets do not have any elements. In logic circuit the OR gate is "true" when one value is true, only will be false when both elements are false.



Truth Table OR		
A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	1

Figure 2.- OR gate and its equivalent in set theory, the disjunction with union





Truth Table NOT	
A	-A
0	1
1	0

Figure 3.- NOT gate and his equivalent in set theory, the negation with complement

The figure 3 shows the equivalent circuit of the complement set. If the element is not inside of the set *A* then is in the complement of *A*. In logic mathematic this is the same to NOT gate, the definition of the complement say:

$$A^c = \{x \notin A\} \quad (3)$$

Where *x* is an element that belongs to set *A*.

III. DISCUSSION (CASE OF STUDY)

We have three sets *A*, *B* and *C*. The *A* set has an intersection with the *B* set but they do not have intersectione with the *C* set, then we have the relation of the equations (4) and (5).

$$x \notin C \wedge (x \in A \wedge x \in B) \Rightarrow x \in C^c \wedge (A \wedge B) \quad (4)$$

$$x \in C^c \wedge (A \wedge B) \Rightarrow x \in \{C^c \cap (A \cap B)\} \quad (5)$$

When the *B* set has an intersection with the *C* set but does not have intersection with the *A* set they have the relations of the equations (6) and (7).

$$x \notin A \wedge (x \in B \wedge x \in C) \Rightarrow x \in A^c \wedge (B \wedge C) \quad (6)$$

$$x \in A^c \wedge (B \wedge C) \Rightarrow x \in \{A^c \cap (B \cap C)\} \quad (7)$$

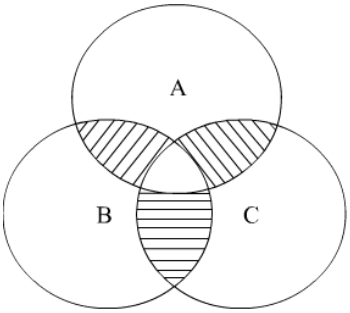


Figure 4: Model that represents the problem of the chat room between three persons. *A*, *B* and *C* are persons represent by sets.

If the *A* set has an intersection with the *C* set but they do not have any intersection with the *B* set then the relations are showed in the equations 8 and 9.

$$x \notin B \wedge (x \in A \wedge x \in C) \Rightarrow x \in B^c \wedge (A \wedge C) \quad (8)$$

$$x \in B^c \wedge (A \wedge C) \Rightarrow x \in \{B^c \cap (A \cap C)\} \quad (9)$$

This problem is structured in logic form like this: We have three persons *A*, *B*, *C* and three conditions:

1) The person *A* wants to connect by chat with the person *B* but they cannot chat with person *C* for the moment (excluding C).

2) The person *B* is connected with person *C* and they do not want to chat with the person *A* for the moment (elimination A).

3) The last case is when A and *C* are connected excluding to the person B.

This case is modeled in set theory using Venn diagram like shows in figure 4. The intersections represent a path of communication between two people.

The set explanation of the problem analyzed is the equation (10).

$$[(A \cap B) - C] \cup [(B \cap C) - A] \cup [(A \cap C) - B] \quad (10)$$

So using a table of equivalences [17-18] into the set theory we have that:

$$x \in (A - B) \Rightarrow x \in (A / B) \Rightarrow x \in A \ y \ x \notin B \quad (11)$$

$$x \in A \ y \ x \notin B \Rightarrow x \in A \ y \ x \in B^c \quad (12)$$

$$x \in A \ y \ x \in B^c \Rightarrow x \in (A \cap B^c) \quad (13)$$

because they are an equivalent form then we can replace:

$$\left[(X \cap Y) - Z\right] = \left[(X \cap Y) \cap Z^c\right] \quad (14)$$

The representation of this equation into circuit logic is shows in figure 5.

So, applying the equation (14) into (10) then we obtain the equation (15).

$$\left[(A \cap B) \cap C^c\right] \cup \left[(B \cap C) \cap A^c\right] \cup \left[(A \cap C) \cap B^c\right] \quad (15)$$

Taking the methodology proposal in this paper then $\left[(A \cap B) \cap C^c\right]$ can changed for the equivalent circuit (figure 5).

Making the truth table for sets is a combination of A, B and C, the three persons, the solution *X* is 1 when there are communication between two persons and 0 when only one person is connected or the three can chat together at the same time.

The final circuit and the truth table are showed in the figure 6. In this table we can see that only the result *X* is true when *A* and *B*, or *B* and *C*, or *A* and *C* are connected, in other combinations the result are false, this correspond with de equation 15.

This gate diagram can be converter in an electric circuit in breadboard (see figure 7) using TTL gates, the results are expressed in the final gate with a led. If the led is turn on then we have a connection between two sets or persons, if the led is turn off then we have a condition not allowed.

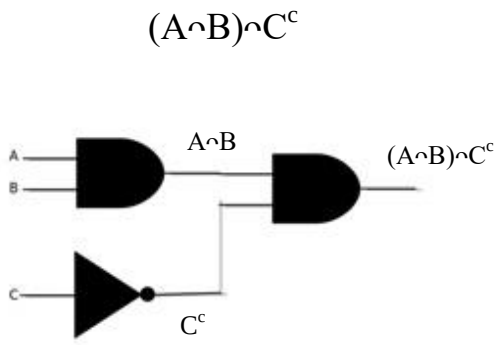


Figure 5: Representation of sets intersection and complement in logic gates.

IV. COLLABORATIVE STUDIES

F or the collaborative education we assign roles to the student, one team is constituted with three persons, the first is called “the checker”, his role is to take the set theory equation and create the equivalent basic logic form. Each checker of different teams works together for this goal.

The second is called “the modeller”, his role is to create the truth table with all possible combinations and solutions. Again, each modeller of different team works for make these solutions.

The third and the last member is called “the constructor”, his role is create the logic diagram based in TTL gates helped with the other constructors.

Truth Table OR			
A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

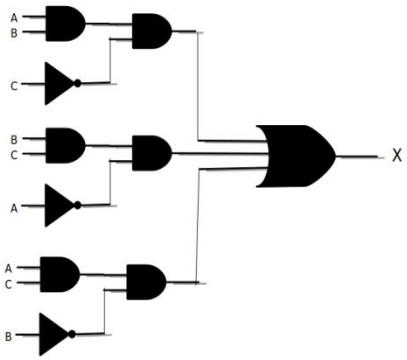


Figure 6. Final truth table and gates diagram with the solution of the case of study. *A*, *B* and *C* are sets or persons, *X* is the final result obtained with this relation.

The figure 7 has this distribution of the students for make one team of checker, modeller and builder and how they interact with the other members of the different teams.

Finally the teams are regrouped with one checker, one modeller and one builder, bring the results obtained previously and together build the circuit into the breadboard following the diagrams of the members of his team (figure 8).

For last, the team test in their circuit probing all combinations of the truth table previously obtained.

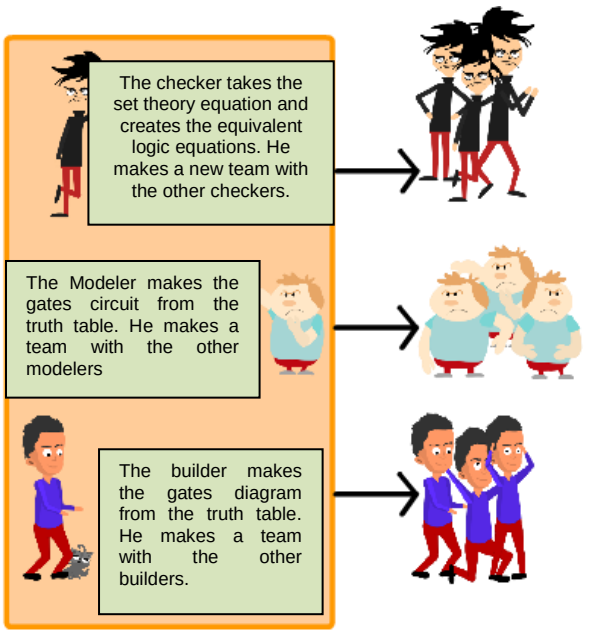


Figure 7.- Teams of the collaborative education. One team has a CHECKER, a MODELLER, and a BUILDER.

The student experience about this method was presented in the table 1. Before of the collaborative education the most students do not understood the problems of the set theory (92%), after that the 90% could understand better this concepts.

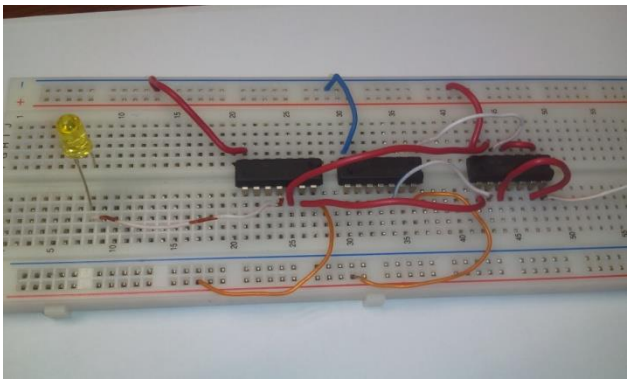


Figure 8: Final design of the circuit

The student perceptions when they interact with the other members of the other teams helped to solve the problem studied of the set theory (87%).
Finally, this new method of to learn set theory of a point of view of digital circuits liked very much to the students, they like the role proposed by the teacher (the checker, the modeller and the builder) and the interactions with his corresponding members of the other teams. The most of the student think that set theory is very interesting and important and is not boring after of this collaborative education studies.

Table 1
Student responses to questions and statements from the collaborative education

Topic	Agreement	Undecided	Disagreement
I understand the problem of set theory before of the study	8%	0%	92%
I understand the problem of set theory after of the study	90%	6%	4%
The collaborative method and the interaction with the other teams made easy the learning.	87%	13%	0%
I liked the set theory before of the collaborative studies	21%	0%	79%
I like the se theory after o the collaborativ studies	94%	1%	5%

V. CONCLUSION

We create a new method for to teach a set theory for engineers. The method is based in the circuit theory, the set theory and the logic gates. Is a novel technique and is based in the collaborative education. Permit to every one of the team have a role and learn how to work collaboratively, like a brick or a little piece of the society. Each student has a principal role in his job and is motivated to understand his learning with a better knowledge of the set theory without the mathematic complexity.
The student perception about the set theory was indifferent previous of this studies. After of this, they liked very much and are most interesting in to know about this mathematic area.

VI. GRATITUDES

Thanks to the students of the Biotechnology Engineering and the Computer Systems Engineering of the Universidad Politécnica Metropolitana de Puebla for this studies and the students of the Instituto Tecnológico Superior de Atlixco and the students of the Instituto Intercanadiense de Puebla.

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VIII. BIOGRAPHY

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